Toets Foutenanalyse 22-1-2012 Uitwerking

Opgave 1

a) $R = \alpha T + \beta T^2 = 0{,}034 \cdot 81 + 0{,}00003(81)^2 = 2{,}754 + 0{,}19683 = 2{,}95083\ \Omega$

b) $\Delta R = \sqrt{\left(\frac{\partial R}{\partial T}\right)^2 (\Delta T)^2} = \frac{\partial R}{\partial T}\Delta T = (\alpha + 2\beta T)\Delta T = (0{,}034 + 2 \cdot 0{,}00003 \cdot 81)0{,}1 = 0{,}003886 \approx 0{,}004\ \Omega$

c) $z = \frac{2{,}952 - 2{,}95083}{0{,}003886} = 0{,}30108$ Aflezen uit de tabel geeft $O = 0{,}1183$, d.w.z.

$P(R > 2{,}952) = 0{,}5 - 0{,}1183 = 0{,}3817$, d.w.z. ca. 38,2%

Opgave 2

a) $\int_{-\infty}^{\infty} Axdx = 1 \ \rightarrow\ A\int_0^4 xdx = \frac{A}{2}[x^2]_0^4 = \frac{A}{2} \cdot 16 = 8A = 1 \ \rightarrow A = \frac{1}{8}$

b) $\bar{x} = \int_{-\infty}^{\infty} xAxdx = \frac{1}{8}\int_0^4 x^2dx = \frac{1}{8\cdot3}[x^3]_0^4 = \frac{64}{8\cdot3} = \frac{8}{3}$

c) $\overline{x^2} = \int_{-\infty}^{\infty} x^2Axdx = \frac{1}{8}\int_0^4 x^3dx = \frac{1}{8\cdot4}[x^4]_0^4 = \frac{4^4}{8\cdot4} = 8$

$\sigma = \sqrt{\overline{x^2} - \bar{x}^2} = \sqrt{8 - \left(\frac{8}{3}\right)^2} = \sqrt{\frac{8}{9}} = \frac{2}{3}\sqrt{2}$

d) $P = \int_{\bar{x}-\sigma}^{\bar{x}+\sigma} Axdx = \frac{1}{8}\int_{\frac{8}{3}-\frac{2}{3}\sqrt{2}}^{\frac{8}{3}+\frac{2}{3}\sqrt{2}} xdx = \frac{1}{16}[x^2]_{\frac{8}{3}-\frac{2}{3}\sqrt{2}}^{\frac{8}{3}+\frac{2}{3}\sqrt{2}} = \frac{1}{16}\left\{\left(\frac{8}{3} + \frac{2}{3}\sqrt{2}\right)^2 - \left(\frac{8}{3} - \frac{2}{3}\sqrt{2}\right)^2\right\} =$

$\frac{1}{16}\left\{\left(\frac{8}{3}\right)^2 + \frac{32}{9}\sqrt{2} + \left(\frac{2}{3}\sqrt{2}\right)^2 - \left(\left(\frac{8}{3}\right)^2 - \frac{32}{9}\sqrt{2} + \left(\frac{2}{3}\sqrt{2}\right)^2\right)\right\} = \frac{1}{16}\left\{\frac{64}{9}\sqrt{2}\right\} = \frac{4}{9}\sqrt{2} \approx$

0,628, d.w.z. ca. 62,8%

Opgave 3

a) $\bar{t} = \frac{1}{N}\sum_{i=1}^{N} t_i = \frac{1}{5}(6{,}17 + 6{,}13 + 6{,}23 + 6{,}12 + 6{,}16) = 6{,}162\ s$

b) $s = \sqrt{\frac{\sum(t_i - t)^2}{N-1}} = \sqrt{\frac{(6{,}17-6{,}162)^2+(6{,}13-6{,}162)^2+(6{,}23-6{,}162)^2+(6{,}12-6{,}162)^2+(6{,}16-6{,}162)^2}{4}} =$

$\sqrt{\frac{(0{,}008)^2+(-0{,}032)^2+(0{,}068)^2+(-0{,}042)^2+(0{,}002)^2}{4}} = \sqrt{\frac{0{,}00748}{4}} = 0{,}0432 \approx 0{,}05\ s$

c) $s_{\bar{t}} = \frac{s_t}{\sqrt{N}} = \frac{0{,}0432}{\sqrt{5}} = 0{,}0193 \approx 0{,}02\ s$; $\bar{t} = (6{,}16 \pm 0{,}02)\ s$

Opgave 4

a) $\bar{V} = \frac{w_1V_1 + w_2V_2}{w_1 + w_2}$ met $w_1 = \frac{1}{0{,}005^2} = 40000$ en $w_2 = \frac{1}{0{,}002^2} = 250000$

$\bar{V} = \frac{40000 \cdot 1{,}003 + 250000 \cdot 1{,}009}{290000} = \frac{292370}{290000} \approx 1{,}0081724\ m^3$

b) $s_{\bar{V}} = \sqrt{\frac{1}{w_1 + w_2}} = \sqrt{\frac{1}{290000}} = 0{,}0018569 \approx 0{,}002\ m^3$

$\bar{V} = (1{,}008 \pm 0{,}002)m^3$

Opgave 5

a) Met behulp van de tabel volgt :

$u\ (cm)$	$F(N)$	$uF(cmN)$	$u^2(cm^2)$
2	1,2	2,4	4
4	2,3	9,2	16
6	3,7	22,2	36
8	4,7	37,6	64

$$\overline{uF} = \frac{1}{4}(2{,}4 + 9{,}2 + 22{,}2 + 37{,}6) = 17{,}85\ cmN$$

$$\overline{u^2} = \frac{1}{4}(4 + 16 + 36 + 64) = 30\ cm^2$$

$$k = \frac{\overline{uF}}{\overline{u^2}} = \frac{17{,}85}{30} = 0{,}595\ Ncm^{-1}$$

b) Uit de tabel volgt: $\sum_{i=1}^{N} D_i^2 = 0{,}027$

$u\ (cm)$	$F(N)$	$D_i(N)$	${D_i}^2(N^2)$
2	1,2	0,01	0,0001
4	2,3	-0,08	0,0064
6	3,7	0,13	0,0169
8	4,7	-0,06	0,0036

$\sigma_F = \sqrt{\frac{1}{N-1}\sum D_i^2} = \sqrt{\frac{0{,}027}{3}} = \sqrt{0{,}009} =$
0,094868

$s_k = \frac{\sigma_F}{\sqrt{N\overline{u^2}}} = \frac{0{,}094868}{\sqrt{4 \cdot 30}} = 0{,}00866 \approx 0{,}009\ Ncm^{-1} \quad \rightarrow \quad k = (0{,}595 \pm 0{,}009)\ Ncm^{-1}$

c)

$$s_k = \sqrt{\sum_{i=1}^{N} \left(\frac{\partial k}{\partial F_i}\right)^2 (\sigma_F)^2} = \sigma_F \sqrt{\sum_{i=1}^{N} \left(\frac{\partial k}{\partial F_i}\right)^2}$$

$$\frac{\partial k}{\partial F_i} = \frac{1}{\overline{u^2}} \frac{\partial \overline{uF}}{\partial F_i} = \frac{1}{\overline{u^2}} \frac{\partial}{\partial F_i} \left\{\frac{1}{N}(u_1F_1 + u_2F_2 + \cdots . + u_NF_N)\right\} = \frac{1}{\overline{u^2}} \left\{\frac{1}{N} u_i\right\}$$

$$s_k = \sigma_F \sqrt{\sum_{i=1}^{N} \left(\frac{1}{\overline{u^2}}\right)^2 \left\{\frac{1}{N} u_i\right\}^2} = \sigma_F \sqrt{\left(\frac{1}{\overline{u^2}}\right)^2 \frac{1}{N^2} \sum u_i^2} = \sigma_F \sqrt{\left(\frac{1}{\overline{u^2}}\right)^2 \frac{1}{N} \overline{u^2}} = \frac{\sigma_F}{\sqrt{N\overline{u^2}}}$$